HISTORIA MATHEMATICA 16 (1989), 316–323

Salomon Bochner as Historian of Mathematics and Science*

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It is impossible for me to convey an appreciation of Bochner and his accomplishment in history of mathematics and science otherwise than in personal terms or to imagine how his writings would read in the eyes of people who did not know him. His judgments were alive with idiosyncrasy. When something that was being presented in seminar pleased him, he would fairly tingle in his place at the table, straining forward, eyes alight, urging the point onward with "Beautiful, very beautiful," half under his breath and perhaps not consciously uttered. Even at those moments we could never feel quite comfortable with our success, for it remained unclear whether we were the ones exciting the response or whether it was Aristotle or Fermat or Leibniz refracted through the glass of our optically uneven sensibilities, now mercifully translucent and too soon again opaque. The opaque interludes were terrible. Bochner's expression would collapse through phases marking disappointment, incomprehension, boredom, and despair until at the worst moments he rescued his sanity, his slightly puckish sanity, by falling asleep. On such occasions he might murmur something at the beginning of the coffee break and go off into the stacks, ostensibly in search of a book, never to return that day. Whether he taught us anything explicitly, I should be hard pressed to say, but certainly his presence, and the dread of provoking his absence, put us on our mettle. That was not his intention, and I think he was unaware of his effect.

His intention was to participate in discourse about the history of science, his violon d'Ingres, which he played to the strains of an inner melody. It happened that a doctoral program in history and philosophy of science got under way at Princeton in 1960. Apparently we did not disgrace ourselves in the first year or two, for Albert Tucker, then Chairman of the Mathematics Department, took me (a fairly junior faculty member) aside one day and intimated that Bochner might be receptive to an invitation to associate himself with our work, but that he was too shy to say so. Bochner, he went on, was not only a leading but a learned mathematician. The former fact had already been born in on me by the subliminal

* This essay was written at the request of mathematical colleagues in 1983, shortly after the death of Salomon Bochner (August 20, 1899–May 2, 1982). It was to have been published with several other memorials concerning the main aspects of his life and career. That project did not go forward. I am grateful to *Historia Mathematica* for allowing me thus to make known the admiration and enjoyment experienced by my students and myself throughout Bochner's participation in our studies during the last 5 years that he served on the Princeton faculty. He retired in 1968 and moved to Rice University, where he was Chairman of the Mathematics Department from 1969 until 1976.

0315-0860/89 $3.00

Salomon Bochner

vibrations that transmit the general consent of a university in matters of reputation. The latter information, such is the frailty of scholarly natures, only increased my apprehensiveness. Would that all unworthy fears turned out to be as groundless, for Bochner's joining the staff of our program in its early years was the making of it.

Bochner had long since been giving himself the pleasure of studying (he might rather have said "savoring," a favorite word) the classics in the history of mathematics and mechanics beginning in antiquity. Since he had been educated in the classical Gymnasium, he knew Greek and Latin no less than modern languages, and he always sought out the earliest edition to be found of any text. His taste in the secondary literature was more haphazard, not to say quixotic. There were penetrating remarks on occasion, but Bochner's judgment of the scholarship of others was not, in my view, always illuminating. On the other hand, he proved to be an excellent judge of the qualitites of our students.

A word first about his vision of the subject—I do not want to call it anything so mundane as a point of view. Emphatically, his was not the attitude that historians of science recognize and tend to deplore as characteristic of scientists and mathematicians: a taste for anecdote, often spiced by a touch of malice; a nose for scandal, particularly in the matter of priorities; an interest in substance only insofar as pieces of past science appear to be approximations, more or less awkward, to what is known now.

Not so Bochner. He was that rare, perhaps that unique mathematician whose historical sensibility was formed, not at bottom by the mathematics he practiced, but by the philological tradition out of which he explored its reaches in civilization. Central European rather than strictly Polish, Bochner was of the last generation of scholars who still incarnated the reality that German universities made of the ideal of culture down to the catastrophes of the 20th century. He sometimes exhibited the classicist's or the humanist's instinct, the reverse of the scientist's, that the ancients, an Aristotle, an Archimedes, a Euclid, were and had to be of greater stature than the moderns. These are only half-heard overtones, however, not symptoms of adulation. More important was the complement. Nowhere in his writings is there the faintest trace of condescension toward scientists or science in the past. When criticism is in order, it is the kind that might be visited on a contemporary.

And yet, we cannot make of Bochner an historian's scientist–historian, treating science in relation to its own time and context rather than as a function of its future. The difference is not only that he betrayed no interest in effects of social and political conditioning, a preoccupation that in the 1970s might have gone to an extreme from which it shows signs of receding. There was more to Bochner's distinctiveness than that. Historians, whatever their persuasion about fundamental factors in the historical process, are interested primarily in development: not just in what the event or, in the case of science, the discovery, theorem, or treatise was in itself, though of course they need to know that, but in how one event followed from and led to others in time and circumstance. In Bochner's perspective all of mathematics wears the appearance not altogether of intellectual contemporaneity, as I was about to say, but of a deeper contemporaneity residing in intuition. His writings, therefore, convey nothing of the process of development of exact science even though they do identify stages through which it has passed. These stages or phases are seen as conditions, however, as a series of states of knowledge with no nexus running between them.

A certain tension, not to say ambivalence, in his outlook is evident in a lengthy essay "How History of Science Differs from Other History" [1966, Chap. 2]. Bochner was widely read in the great historians—Herodotus, Thucydides, Tacitus, Gibbon, Ranke, Mommsen. There are hints that he really thought general history a richer and more rewarding body of subject matter than history of science. He once said so to me. That is not just because general history concerns the whole life of mankind, but because the strict criteria of science impose limits on what can be said about its history. Archimedes was a judge of his own accomplishments in a way that Alexander the Great was not. Science internalizes its own history and in some degree represses it. The historian of science must be circumspect and respect the taboos that are inseparable from its creation and also the success of the outcome that has prevailed. General history is free of such constraints and is inexhaustible. At the same time, Bochner's sense of general history was by no means that of a professional historian. The distinctions on this side are of a different order, however. His was the historiography of Hegel, of the Germanic personification of abstractions, of—not to put too fine a point upon it—Spengler, and no practitioner of history as a succession of mind-sets of the Zeitgeist could have been more out of favor than Spengler among historians for the last half-century, for both professional and political reasons. In Bochner's essays, too, the great historical periods become actors in the drama: the Middle Ages, the Renaissance, the 17th, 18th, and 19th centuries. Each is endowed with aspects of collective intellect and will; thus: "The 17th century was an age of revelation: the 18th century was an age of patristic organization; and the 19th century was an age of canonical legislation. If we dared to continue we might suggest that the 20th century is an age of reformation. . . ." Periods and even sciences become subjects of verbs in the active voice, thus: "The Renaissance did little for physics but much for mathematics," and "Rational Mechanics also gradually introduced the concept of a purely mathematical space which is multi-dimensional . . ." [1966, 180, 220, 246].

Although Bochner conveyed no notion of strictly historical development, in the case of mathematics and mechanics he nevertheless did see growth, a kind of intrinsic maturing of collective powers. These actualizations of a potentiality in the life of the discipline are virtually Aristotelian. They are not explained. They are recognized. They come about "somehow," another favorite word, which Bochner somehow managed to employ with particular precision, and not as a vague gesture pointing away from ignorance. The organic quality of their emergence is further and pervasive witness to the chrysalis of Germanic historical sensibility.

A deeper ambivalence underlies Bochner's uncertainty over the value to be placed upon history of science in relation to general history. What is to be thought of mathematics itself, its very possibility, its claim upon culture, its penetration of other and perhaps one day all formal knowledge? In many passages he refers to mathematics as an esoteric activity, almost as a kind of pastime that by some destiny akin to that of myth imposes its rules upon ever wider sectors of reality. The figure of a game helps resolve a feature of his treatment that might otherwise appear inconsistent with the notion of intellectual contemporaneity and comparability of exact knowledge. For Bochner does feel free to take the Greeks to task for the ultimate sterility of their absorption in geometry, and specifically for their having missed developing a system of spatial and temporal coordinates. They were not inferior thinkers in his eyes. No, they were athletes who refused to modify their strategy and gain what was within their powers. Their failure appeared to Bochner almost "inexcusable" [1966, 52]. A game is to be won.

Apart from that, it is by no means obvious that other elements in the surrounding community have reason to welcome extrapolation of the rules of any such game. Their own pleasure may be spoiled, their autonomy infringed. In fact, however, expansion of mathematics has transpired more by invitation from recipients than by imposition from practitioners. At all events, there is nothing to be done about it. Bochner emphatically did not believe in the possibility, let alone the desirability, of channeling or guiding mathematical investigations. When he spoke of younger generations and the directions they were taking, not all of which were to his taste, he would say, "You cannot prevent them," without even a hint that he wished you could.

The first of Bochner's historical papers, "The Role of Mathematics in the Rise of Mechanics," appeared in 1962. Substituting the word "science" for "mechanics," he also made that the title of his first book, a collection of this and other essays published in the next 4 years. Another volume, *Eclosion and Synthesis, Perspectives on the History of Knowledge* [1969], was written in part out of his participation in the seminars of our Program in History and Philosophy of Science. In his last years at Rice University, Bochner composed further articles and monographs, several of them in the nature of reminiscence. In most instances, the titles of his papers are less indicative of the contents than is customary in historical writing. Instead, each of the ostensible topics serves as the occasion to embark on discussion of certain favorite themes to which he recurs from these various points of departure. It will be more pertinent, therefore, to identify the themes themselves than to attempt a summary of discrete contributions.

The underlying preoccupation, and perhaps the most signal motivation of the entire *oeuvre,* is indicated by the above title as modified to cover the contents of the earlier of the two books. How does it happen that mathematics applies to nature and even to society in such wise that it has become the most powerful strain in science? Ultimately, Bochner regards that, the dominant fact of Western intellectuality, as a mystery which he proposes to bring into relief in various ways but not to dispel. The Greeks, for their part, never developed a mathematical physics. Lacking, in Bochner's view, were an idea of quantity expressible in real numbers, the practice of analysis, and the capacity for abstraction. Only in 17th-century mechanics did the concept of a moment become possible, the real-number product of unlike magnitudes.

At this overt level, these findings of Bochner, and others like them, contain few surprises. The merit and originality of his discussion lie instead in many shrewd remarks in passing on such matters as functionality in Newton's thinking and in general, 18th-century principles of mechanics, the absence of any need for energy considerations before the 19th century, the emergence of distinctions between scalar and vector quantities, the "complexification" of physics through the introduction of complex numbers, and so on. There are many variations on two recurrent themes, first that there is a reciprocity between mathematics and physics, but second that it does not consist in deliberate steps, however successful, to mathematicize physics for specific purposes. Much of the mathematics that has proved most important to physics has originated without any thought of application in the course of purely mathematical investigations. Bochner was particularly intrigued by the frequent recourse in physics to pieces of "pre-fabricated mathematics," resources like the tensor calculus of Ricci and Levi-Civita lying there ready to Einstein's hand.

In my view, the best sustained of Bochner's substantive discussions is the essay "Aristotle's Physics and Today's Physics" [1964] together with the remarks on physical and mathematical space that figure in that paper and are then elaborated in several others. Although his views on the historical process are redolent of 19th-century philosophy, his views on science tend to come fundamentally down to earth. He resonates to Aristotle, not to Plato; to quantum mechanics, not to the mystique of relativity (his essay on Einstein [1979a] borders on the iconoclastic); to Kepler and Newton and Euler, not to Descartes and Leibniz and Kant. Here and there he will indulge philosophers who say things about scientific matters. Their "philosophemes" were another kind of pastime, however, that unlike mathematics came to nothing beyond embroidery. Essentially, he considers that Aristotle's *Physics* is to be read as physics, and not as philosophy. The notion of time as a determinant of motion or change; the correspondence of its oneness to that of the universe; the study of cosmology as an extension of physics; the relation of chance to necessity; *topos* or place involving a notion of spatiality as the setting for a physical system; the explanation of motion by antiperistasis, or action of a medium, as akin to thermodynamical processes; certain of the yes-or-no signals that constitute computerized information—in these and other matters, Bochner finds comparisons between Aristotelian and modern modes of thinking that illuminate the one by the other, without his wanting to say that it is a question of anticipations or reversions. The thought patterns are comparable, that is all.

Eclosion and Synthesis is a rather high-flown title for the second of Bochner's historical volumes [1969]. Perhaps the book may best be described as a tentative sketch for a comparative morphology of two great stages in the evolution of systematic knowledge. Bochner designates by "eclosion" the characteristics of the half-century from 1776 to 1825 and by "synthesis" those of the 20th century, mostly since the 1920s. Clearly, these terms would bear an enormous weight of generalization if they were to be taken literally and exhaustively. I do not think Bochner meant them that way. I think he meant them impressionistically as labels permitting him to gather into two loose bundles reflections upon his reading and experience. The opinions are a good deal more then desultory and a good deal less than systematic. More often than not they amount to insights. For example, the period he calls eclosion had no identity in the historiography of science at the time of his writing. What he ascribes to it is not so much cognitive discovery or conceptual innovation as it is the concretization of the modern learned disciplines in the form of professional entities—physics, chemistry, economics, and others, with emphasis on the very creation of mathematical and theoretical physics. Now, this observation, which Bochner does little more than assert, was right on target. The movement is now called the second scientific revolution, although I doubt that any of the many people working with its problems owe their start to his small, aphoristic book.

The notion of synthesis in the 20th century is less fully circumscribed by dates and less tellingly illustrated by cross-disciplinary comparisons. By it, he means the momentum acquired by modern bodies of knowledge, their tendencies to internalize differences and disputes rather than to allow themselves to be transformed, the accelerating displacement of intellectuality by factuality, and the widening and tightening of the grasp of mathematics. I think the most interesting passages, however, are those conveying Bochner's skepticism, expressed in relation to modern cosmology, about the commitment of Newton and classical physics to the infinity of the universe, and those concerning Cézanne's liberation of artistic perspective from the straitjacket of three-dimensional Euclidean space.

In general, it should perhaps be said in conclusion, Bochner's historical writings are more rewarding for their asides, their irreverences, their glancing observations, than for their arguments, which are too fragmentary, introspective, and elliptical to be often persuasive. I do not mean that as anything but a high compliment. Professional life is stuffed with colleagues ever at our elbows trying to convince us. How much more agreeable on occasion, and how rare, to be intrigued, to be amused, to be startled, to see something differently, however fleetingly. I close with a few examples. Of Euclid and his immunity to human and historical influences: "In short, it is almost impossible to refute an assertion that the *Elements* is the work of an unsufferable pedant and martinet" [1966, 35]. Of rational mechanics versus the rest of physics: "Thus Newton composed not only his formidable *Principia* but also an insinuating *Opticks,* at first not in Latin but in English, so that even poets might read it, as some did" [1966, 221]. And, finally, to the complaint that people missed the chance Leibniz had provided to found mathematical logic in the 18th century:

> To this I wish to say, from hindsight, that, as developments went, everything turned out as well as one could wish for. The 18th century was extremely wise to give priority to constructing a thick basic layer of mathematics and erecting an edifice of rational mechanics; the 19th century was then the readier to initiate a mathematization of physics and of logic and the theoretization of other science. If this kind of rationalization of mine is too crass a case of "being wise after the event," then I wish to observe that, for my part, I have never felt dismay over savoring history, any history, backward through time, in addition to viewing it forward through time. [1966, 101]

SALOMON BOCHNER: WRITINGS ON HISTORY OF SCIENCE AND MATHEMATICS

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